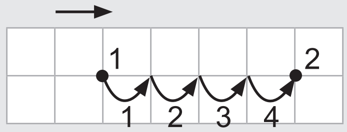
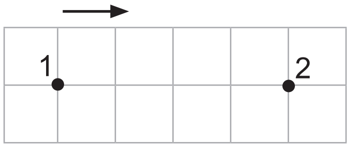
G6-13 Translations

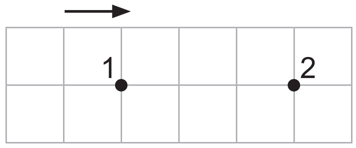
Josh slides a dot from one position to another. To move the dot from position 1 to position 2, Josh slides the dot 4 units right.
In mathematics, slides are called **translations**.



1. How many units **right** did the dot slide from position 1 to position 2?

a) 

_______ units right

b) 

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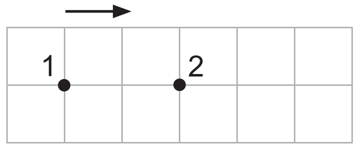
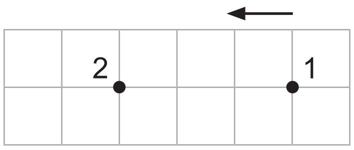
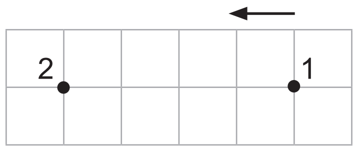
c) 

____

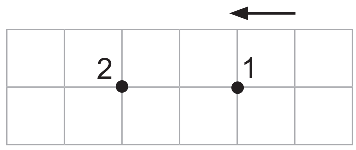
2. How many units **left** did the dot slide from position 1 to position 2?

a) 

_______ units left

b) 

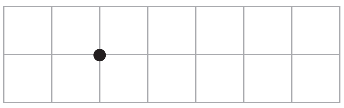
____

c) 

____

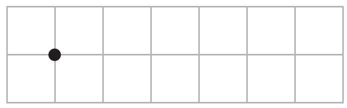
3. Follow the instructions to translate the dot to a new position.

a) 3 units right

L 

b) 4 units left

c) 5 units right

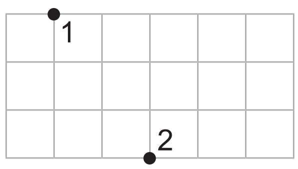
 R

4. Describe the translation of the dot from position 1 to position 2.

a) 

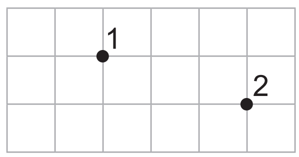
______ units right

______ units down

b) 

______ units right

______ units down

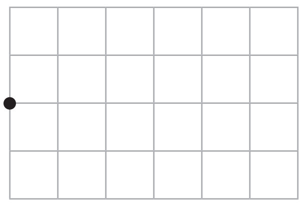
c) 

______ units right

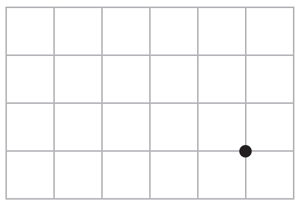
______ unit down

5. Translate the dot.

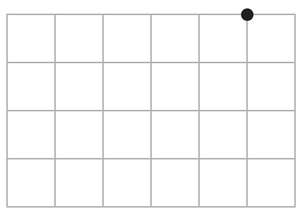
a) 5 units right, 2 units down



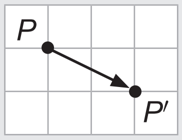
b) 4 units left, 2 units up



c) 3 units left, 4 units down

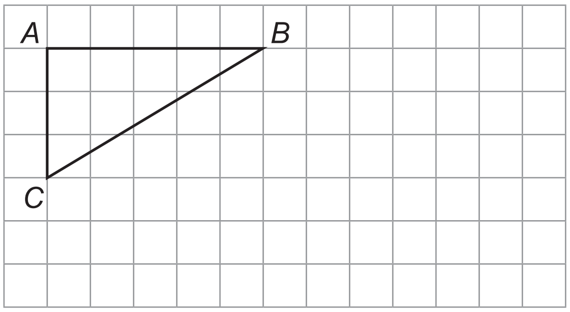


The result of a translation is called the **image under translation**.
You can use the **prime symbol** (′) to label the image.
Example: The image of *P* under translation is *P*′.

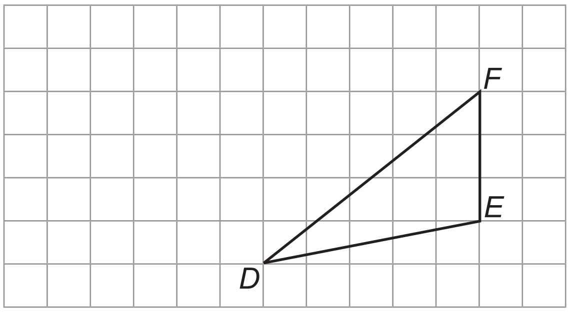


6. a) Use a ruler and protractor to measure the sides and the angles of the triangle.

i)



ii)



$AB =$ ______ mm $\angle A =$ ______

$AC =$ ______ mm $\angle B =$ ______

$BC =$ ______ mm $\angle C =$ ______

$DE =$ ______ mm $\angle D =$ ______

$EF =$ ______ mm $\angle E =$ ______

$DF =$ ______ mm $\angle F =$ ______

b) Translate the triangle by translating the vertices. Use ′ to label the images of the vertices.

i) 5 units right and 2 units down

ii) 4 units left and 1 unit up

c) Measure the sides and the angles of the image.

i) $A'B' =$ ______ mm $\angle A' =$ ______

$A'C' =$ ______ mm $\angle B' =$ ______

$B'C' =$ ______ mm $\angle C' =$ ______

ii) $D'E' =$ ______ mm $\angle D' =$ ______

$E'F' =$ ______ mm $\angle E' =$ ______

$D'F' =$ ______ mm $\angle F' =$ ______

d) What do you notice about the sides and angles of the triangles and their images?

____

7. True or false? If the statement is true, explain why. If the statement is false, draw an example to show it is not true.

a) A triangle and its image under translation are congruent.

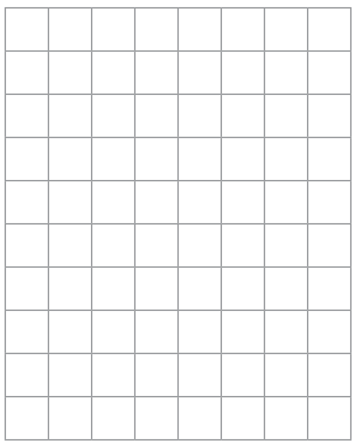
____

____

BONUS ▶ If two triangles are congruent, there is always a translation that takes one of them onto the other.

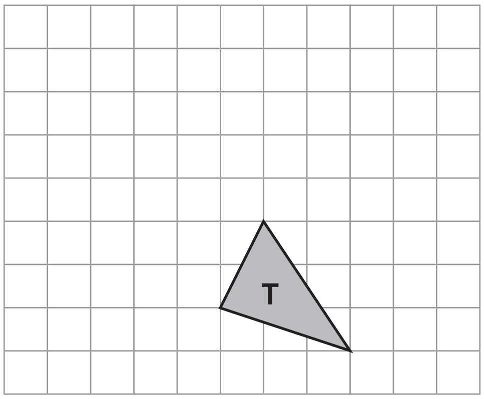
____

____

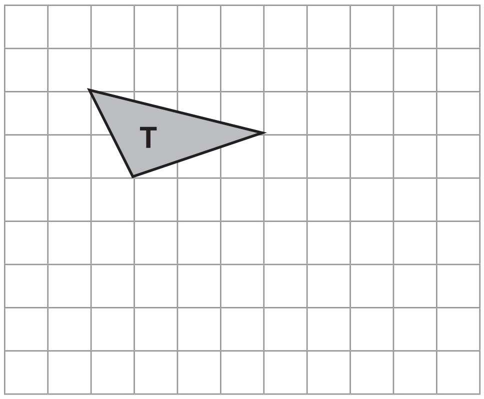


8. a) Translate triangle T as given. Label the image T′. Then translate the image again from T′ to T*.

i) 2 units up and 3 units left, then 1 unit up and 5 units right



ii) 4 units down and 3 units right, then 3 units up and 4 units left



b) Draw arrows joining the corresponding vertices of triangles T and T*.

What do you notice about the direction of the arrows? ______________________________

c) Measure the arrows in millimetres. What do you notice about the length of the arrows? ____

d) Can you use one translation to take triangle T to T*? ______ If yes, describe the translation.

i) ______ units ___________ and ______ units ___________

ii) ______ unit ___________ and ______ unit ___________

9. a) Draw a quadrilateral that is not a rectangle in the shaded zone on the grid. Label it Q.

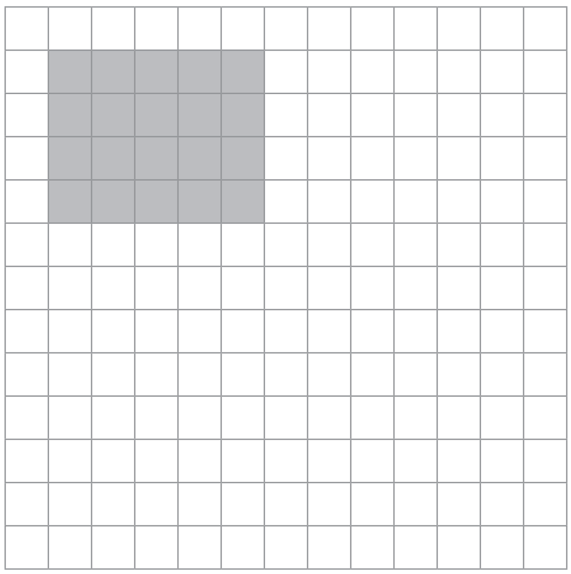
b) Predict the result of combining two translations:

Q to Q′: 6 units right and 3 units down

Q′ to Q*: 4 units left and 4 units down

Q to Q*: ______ units ___________ and ______ units ___________

c) Translate Q to Q′ and Q′ to Q* to check your prediction. Was your prediction correct? ______



10. Jax thinks translating a shape 3 units up and 4 units left, then 4 units right and 3 units down results in the original shape. Is he correct? Explain.

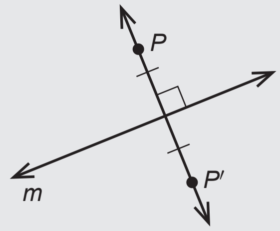
G6-14 Reflections

To **reflect** a point *P* in a **mirror line** *m*:

Step 1: Draw a line through *P* perpendicular to *m*. Extend it beyond *m*.

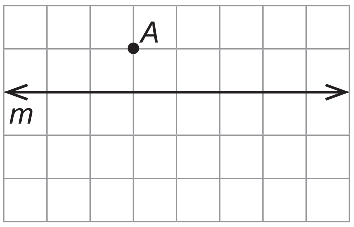
Step 2: Measure the distance from *P* to *m* along the perpendicular.

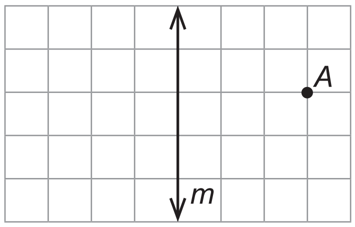
Step 3: Mark the point *P′* on the perpendicular on the other side of *m* so that *P* and *P′* are the same distance from the mirror line *m*.

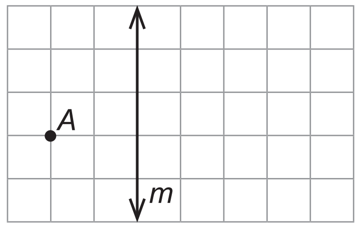


Point *P′* is the **mirror image** of *P*. Mathematicians say that *P′* is the **image of *P* under reflection** in the line *m*.

1. Count the grid squares to reflect point *A* in the given line.

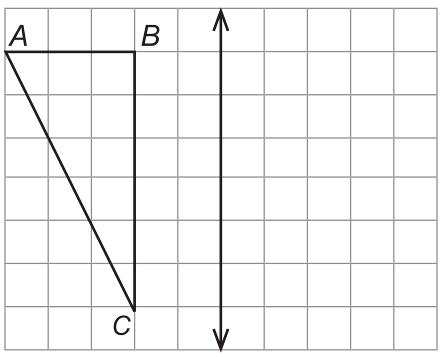
a) 

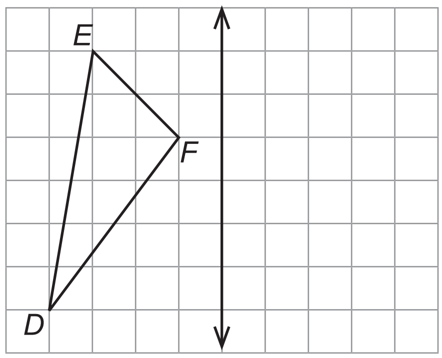
b) 

c) 

To reflect a shape in a mirror line, reflect the shape's vertices and then join the images of the vertices.

2. a) Use a ruler and protractor to measure the sides and the angles of the triangle.

i) 

ii) 

AB = ______ mm ∠*A* = ______

AC = ______ mm ∠*B* = ______

BC = ______ mm ∠*C* = ______

DE = ______ mm ∠*D* = ______

EF = ______ mm ∠*E* = ______

DF =______ mm ∠*F* = ______

b) Reflect each triangle in the given line. Use ′ to label the images of the vertices.

c) Measure the sides and the angles of each image.

i) *A′B′* = ______ mm ∠*A′* = ______

A′C′ = ______ mm ∠*B′* = ______

B′C′ = ______ mm ∠*C′* = ______

ii) *D′E′* = ______ mm ∠*D′* = ______

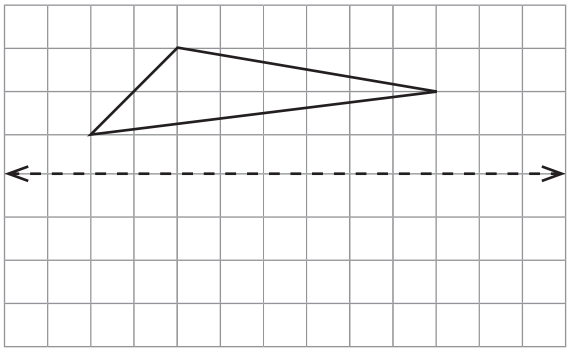
E′F′ = ______ mm ∠*E′* = ______

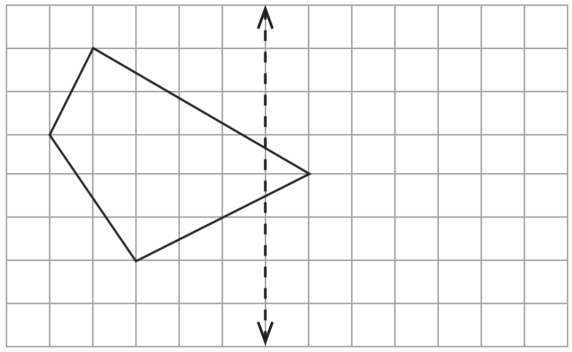
D′F′ = ______ mm ∠*F′* = ______

d) What do you notice about the sides and the angles of each triangle and its image? ______________

Do reflections take triangles to congruent triangles? __________

3. a) Reflect the polygon in the given mirror line.

i) 

ii) 

b) Draw a line segment between each vertex in part a) and its image. What do you notice about the line segments? ____________________________________

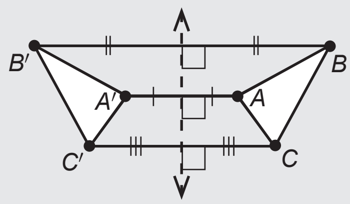
The **midpoint** of a line segment is the point halfway between the end points of the line segment.

c) On the grids above, mark the midpoints of the line segments you drew in part b).
What do you notice about the midpoints? __________________________

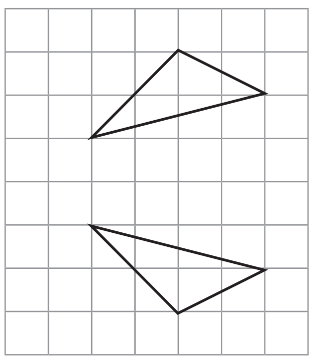
The shapes *ABC* and *A′B′C′* are mirror images of each other when:

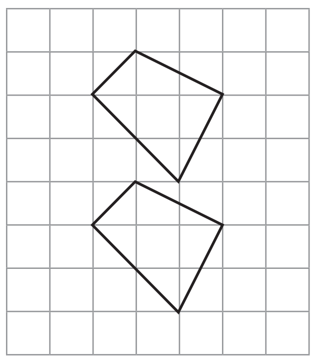
- line segments between each vertex and its possible image are parallel; and
- all the midpoints of these line segments fall on the same perpendicular line.

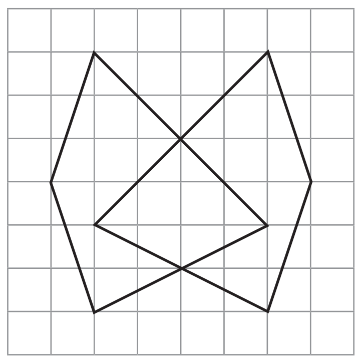
Note: The line segments between the vertices have different lengths.



4. a) Draw line segments between the vertices of the shape and their images.

i) 

ii) 

BONUS ▶ 

b) Find the midpoint of each line segment you drew in part a). Are the midpoints on the same line?

c) Are the shapes reflections of each other? How do you know?

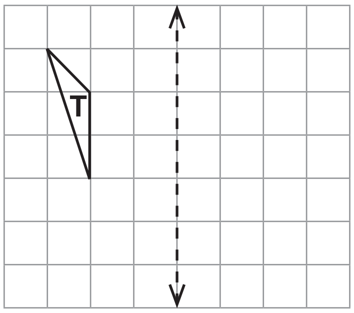
BONUS ▶ If your answer in part c) was "no" for any pair of shapes, identify the transformation that takes one shape into the other.

5. Fill in the table to summarize what happens to a shape that is reflected. What happens when a shape is translated?

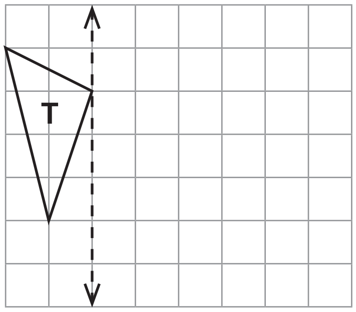
Transformation	Lengths of Sides	Sizes of Angles	Orientation
Reflection			
Translation			

6. a) Reflect triangle T in the mirror line. Label the image T′.

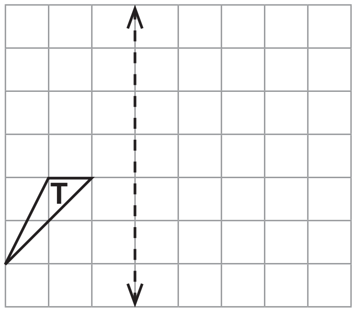
i)



ii)



iii)



b) Translate T′ as given. Label the image T*.

i) 3 units down ii) 4 units right iii) 3 units up and 2 units right

c) Draw the line segments joining each vertex in T to its image in T*. Are the line segments parallel?

i) ________________ ii) ________________ iii) ________________

d) Are the line segments you drew in part c) equal?

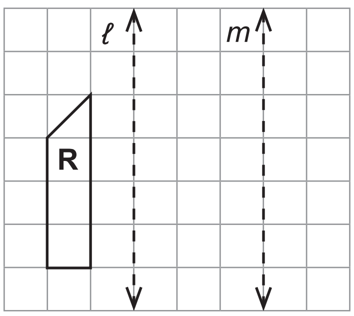
i) ________________ ii) ________________ iii) ________________

e) If possible, draw the translation arrow or the mirror line from T to T*.

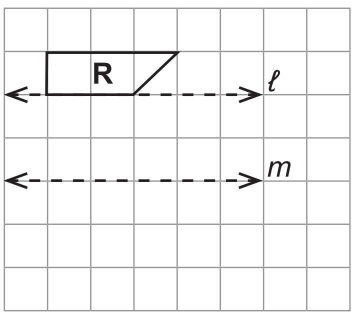
f) Are triangles T and T* congruent? How do you know?

7. a) Reflect the trapezoid R in line ℓ. Label the image R′.

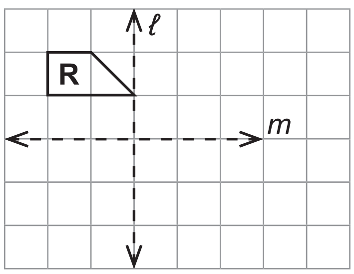
i)



ii)



BONUS ▶



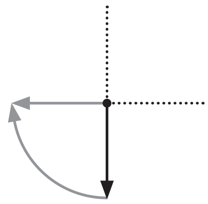
b) Reflect R′ in line *m*. Label the image R*.

c) Is there a reflection or a translation that takes R to R*? If yes, describe it.

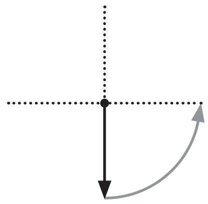
Rotations

1. From the dark arrow, draw an arc showing the direction of the given 90° turn. Draw the arrow after turning.

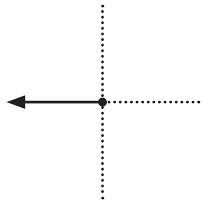
a) clockwise



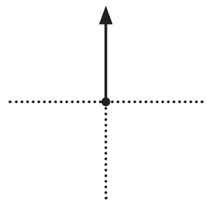
b) counter-clockwise



c) clockwise



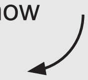
d) counter-clockwise



To **rotate** point *P* around point *O* 90° clockwise:

Step 1: Draw line segment *OP*. Measure its length.

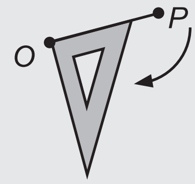
Step 2: Draw an arc clockwise to show the direction of rotation.



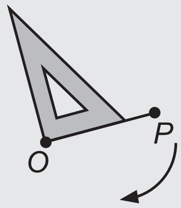
Step 3: Place a set square so that:

- the arc points at the diagonal side,
- the right angle is at point *O*, and
- one arm of the right angle aligns with *OP*.

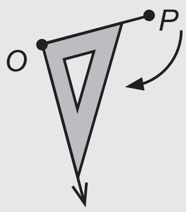
like this



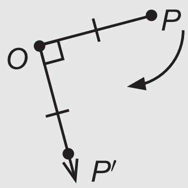
not like this



Step 4: Draw a ray from point *O* along the side of the square corner.

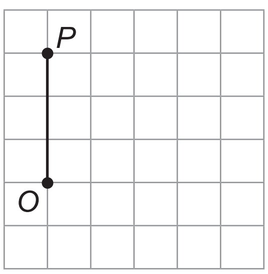


Step 5: On the new ray, measure and mark the image point *P′* so that $OP' = OP$.

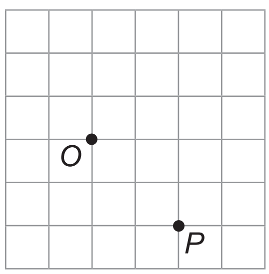


2. Rotate point *P* 90° around point *O* in the direction given. Label the image *P′*.

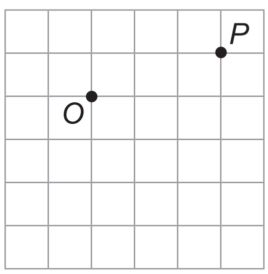
a) clockwise



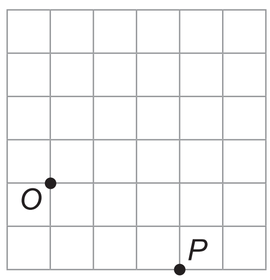
b) counter-clockwise



c) clockwise



d) counter-clockwise



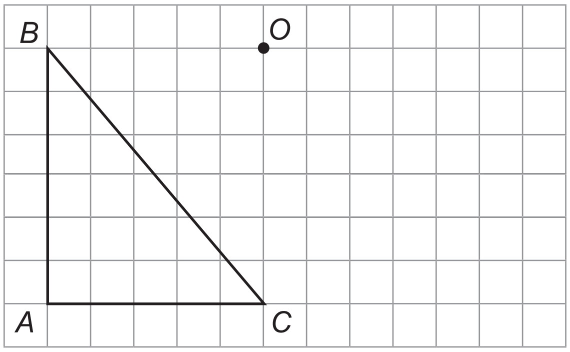
3. Is point *P′* in Question 2 always on a grid line intersection? ______ If not, fix your mistake.

To rotate a shape around point O, rotate the shape's vertices and join the images of the vertices.

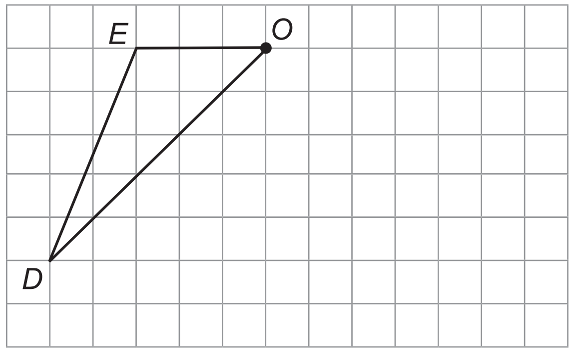
The point O is called the **centre of rotation**. The centre of rotation can be outside, inside, or on a side of the shape. The centre of rotation is the only **fixed point** during a rotation; it does not move.

4. a) Measure the sides and the angles of the triangle.

i)



ii)



$AB =$ ______	$\angle A =$ ______	$DE =$ ______	$\angle D =$ ______
$AC =$ ______	$\angle B =$ ______	$EO =$ ______	$\angle E =$ ______
$BC =$ ______	$\angle C =$ ______	$DO =$ ______	$\angle O =$ ______

b) Rotate the triangle 90° counter-clockwise around point O. Use $'$ to label the vertices of the image.

c) Measure the sides and the angles of the image.

i) $A'B' =$ ______	$\angle A' =$ ______	ii) $D'E' =$ ______	$\angle D' =$ ______
$A'C' =$ ______	$\angle B' =$ ______	$E'O =$ ______	$\angle E' =$ ______
$B'C' =$ ______	$\angle C' =$ ______	$D'O =$ ______	$\angle O =$ ______

d) What do you notice about the sides and the angles of each triangle and its image? ____________

Does rotation take polygons to congruent polygons? ____________

5. True or false? If the statement is true, explain why. If the statement is false, draw an example showing it is false.

a) A polygon and its image under rotation are congruent.

b) If two polygons are congruent, there is always a rotation that takes one polygon onto the other.

6. Fill in the table to summarize. What happens to a polygon that is reflected? Translated? Rotated?

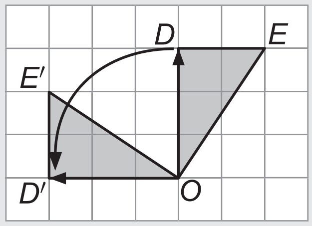
Transformation	Lengths of Sides	Sizes of Angles	Orientation
Reflection			
Translation			
Rotation			

You can rotate a triangle 90° using a grid instead of a set square.

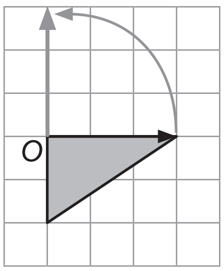
Triangle *OED* has a horizontal side 2 units long and a vertical side 3 units long.

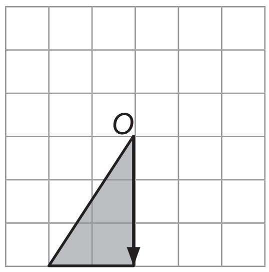
Rotations take triangles to congruent triangles. A rotation of 90° takes horizontal lines to vertical lines and vertical lines to horizontal lines.

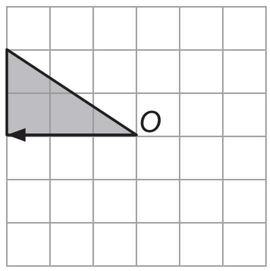
Triangle *OE′D′* has a horizontal side 3 units long and a vertical side 2 units long.

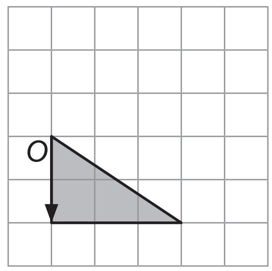


7. Rotate the triangle 90° counter-clockwise around point *O*. Start with the side marked by an arrow. Hint: Note the direction first.

a) 

b) 

c) 

d) 

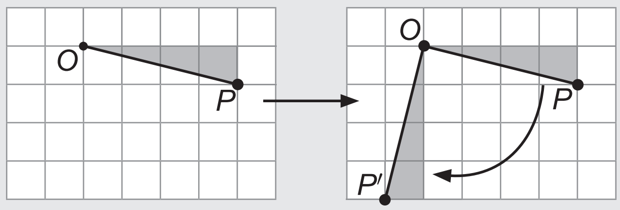
To rotate a point on a grid 90° clockwise around the point *O*:

Step 1: Draw line segment *OP*.

Step 2: Shade a right triangle with *OP* as one side.

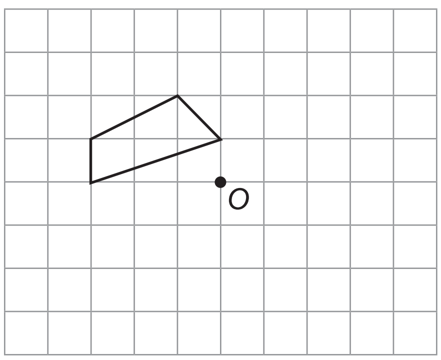
Step 3: Rotate the triangle 90° clockwise around *O*.

Step 4: Mark the image point.

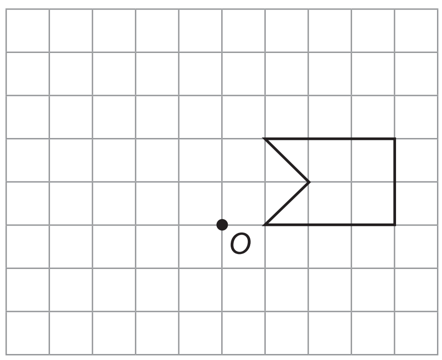


8. Imagine the triangles to rotate the vertices of the polygon around the point *O*. Join the vertices to create the image of the polygon.

a) 90° clockwise



b) 90° counter-clockwise



BONUS ▶ Use a ruler to draw a scalene obtuse triangle *ABC*. Find the midpoint of side *AC* and label it *M*. Rotate triangle *ABC* 180° clockwise around point *M*. What type of quadrilateral do triangle *ABC* and its image make together? Explain.

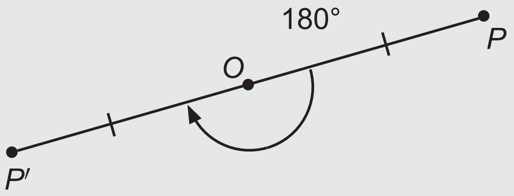
G6-16 More Rotations

To rotate point P around point O 180° clockwise:

Step 1: Draw line segment OP. Measure its length.

Step 2: Extend OP beyond point O.

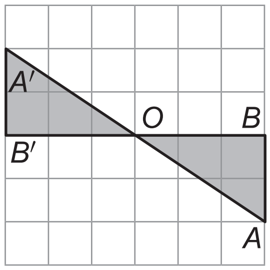
Step 3: Mark the point P' so that $OP' = OP$.



1. Triangle $A'OB'$ is the image of triangle AOB under a 180° clockwise rotation around point O.

a) Triangle AOB has a horizontal side ______ units long and a vertical side ______ units long.

Triangle $A'OB'$ has a horizontal side ______ units long and a vertical side ______ units long.



b) Write "horizontal" or "vertical" to complete the sentence.

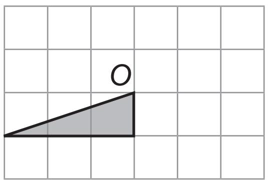
A 180° rotation clockwise or counter-clockwise takes horizontal lines to ________________ lines and vertical lines to ________________ lines.

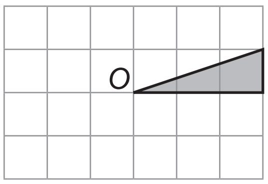
BONUS ▶ Explain why a rotation of 180° clockwise produces the same result as a rotation 180° counter-clockwise around the same centre.

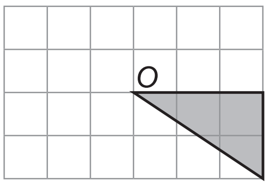
____

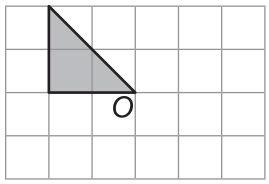
____

2. Rotate the triangle 180° clockwise or counter-clockwise around point O. Start with a horizontal or a vertical side.

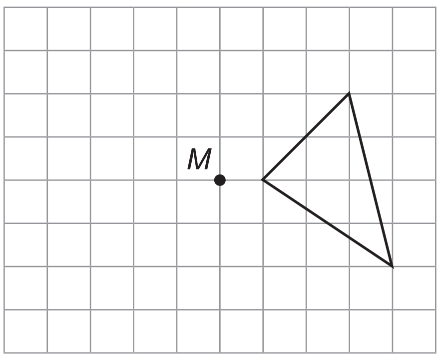
a) 

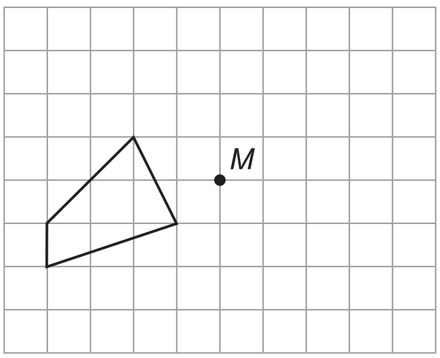
b) 

c) 

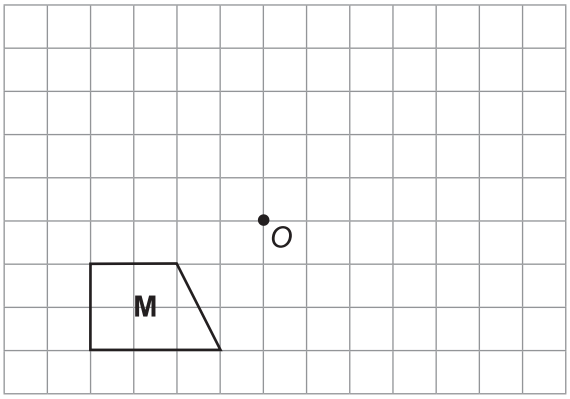
d) 

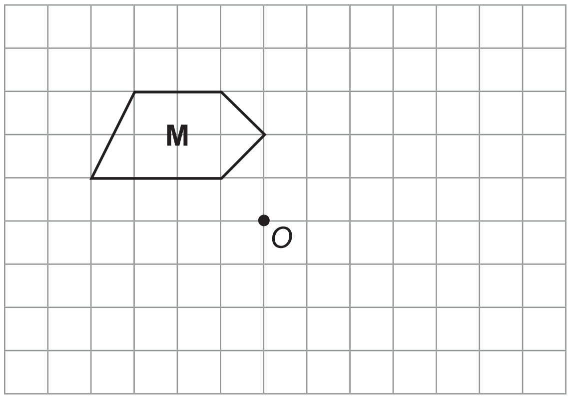
3. Rotate the vertices of the polygon 180° clockwise around point M. Join the vertices to create the image of the polygon.

a) 

b) 

4. a) Rotate polygon M 90° clockwise around point *O*. Label the image M′.

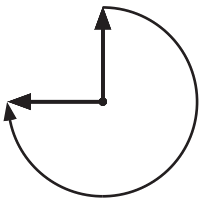
i) 

ii) 

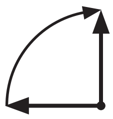
b) Rotate polygon M′ 90° clockwise around point *O*. Label the image M*.

c) Which rotation around point *O* takes polygon M to polygon M*? ______________________

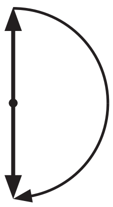
5. How much did the thick arrow turn? Write “90°,” “180°,” or “270°.”

a) 

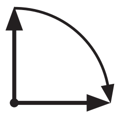
270° clockwise

b) 

________ clockwise

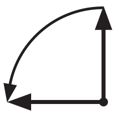
c) 

________ clockwise

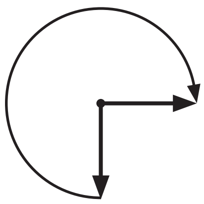
d) 

________ clockwise

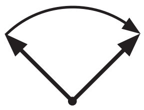
6. How did the thick arrow turn? Use CW for clockwise and CCW for counter-clockwise.

a) 

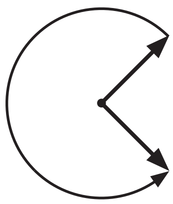
90° CCW

b) 

____

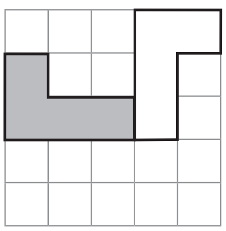
c) 

____

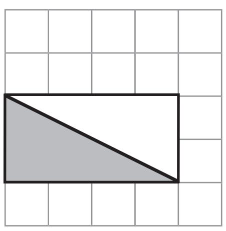
d) 

____

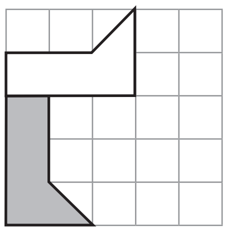
7. Was the grey shape rotated 90° CW, 90° CCW, or 180° CW or CCW to get the white shape? Write the amount and direction of rotation.

a) 

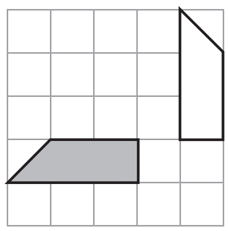
____

b) 

____

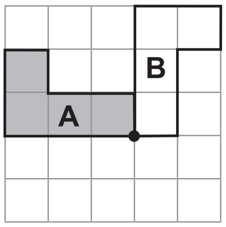
c) 

____

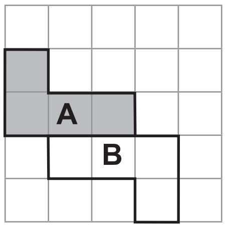
d) 

____

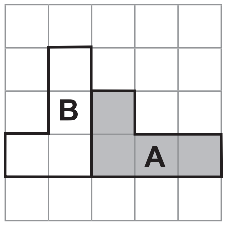
8. Shape B is the image of Shape A under rotation. Mark the centre of rotation and describe the rotation.

a) 

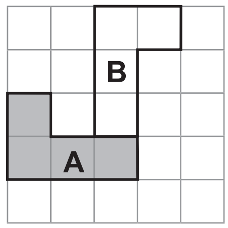
90° CW

b) 

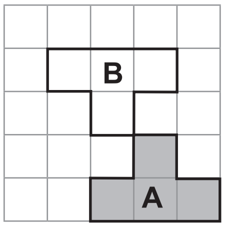
______

c) 

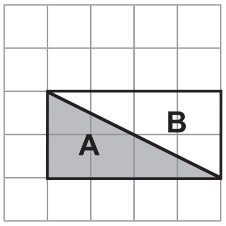
______

d) 

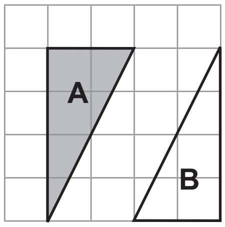
______

e) 

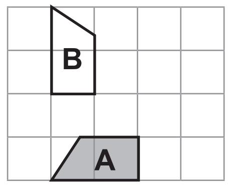
______

f) 

______

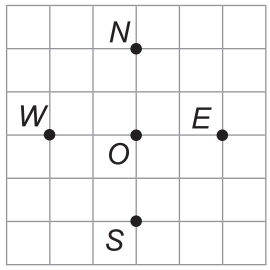
g) 

______

BONUS ▶ 

______

9. Dory rotates point *N* around point *O* as given. What is the image point?



a) 90° CW, then another 90°CW: ______

b) 90° CW, then 180°CW: ______

c) 180° CCW, then another 180°CCW: ______

d) 180° CW, then 90°CW: ______

e) 90° CW, then 90°CCW: ______

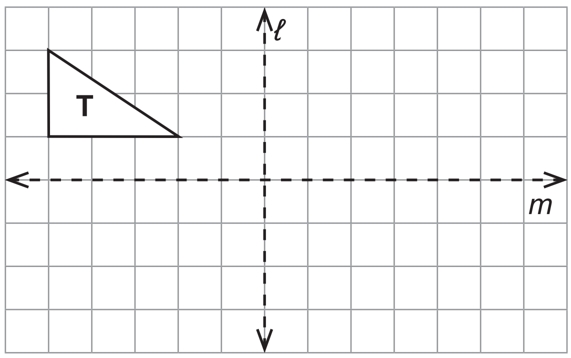
10. a) Reflect triangle T in line ℓ. Label the image T′.

b) Reflect T′ in the line *m*. Label the image T*.

c) Reflect T in the line *m*. Label the image T″.

d) Reflect T″ in the line ℓ. Label the image T**.

e) What do you notice about T* and T**?



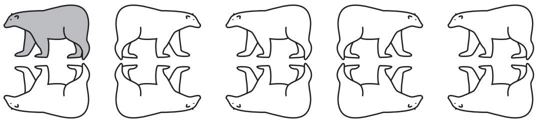
____

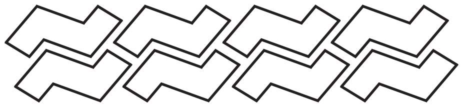
f) Which transformation takes T to T*? Draw the translation arrow, the mirror line, or the centre of rotation and describe the transformation.

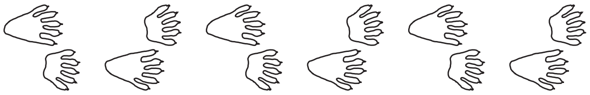
____

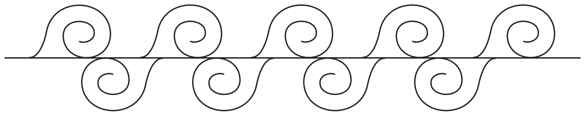
G6-17 Designs and Transformations

1. Shade the smallest part that is transformed to create the pattern.

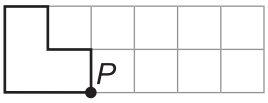
a) 

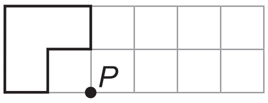
b) 

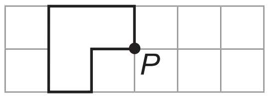
c) 

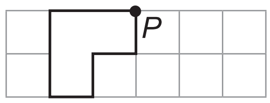
d) 

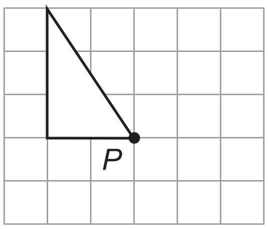
2. Rotate the polygon around point *P* as given.

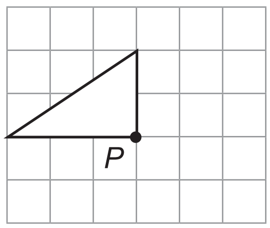
a) 90° CW


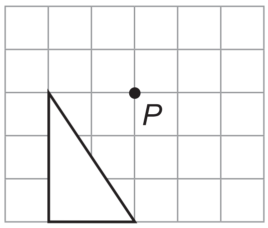
b) 90° CW


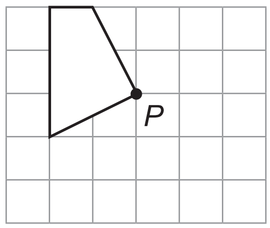
c) 180° CW


d) 90° CCW


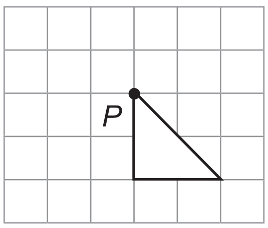
e) 90° CCW


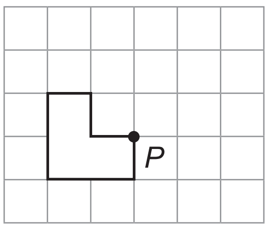
f) 180° CW


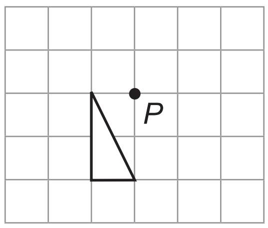
g) 90° CW


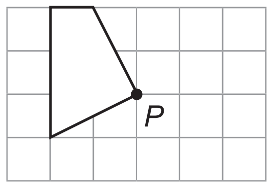
BONUS ▶ 90° CCW


3. Create a design by rotating the polygon repeatedly 90° clockwise around point *P*.

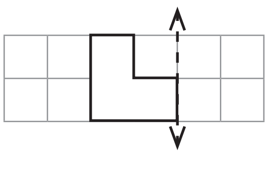
a) 

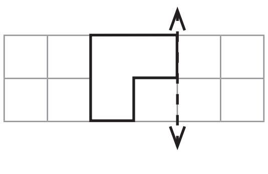
b) 

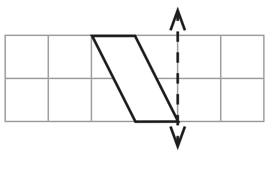
c) 

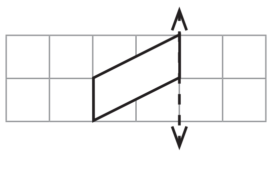
BONUS ▶


4. Reflect the polygon in the mirror line.

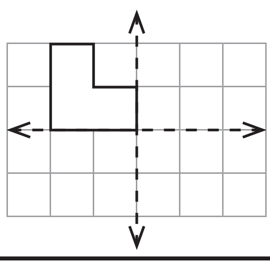
a) 

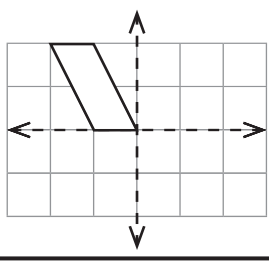
b) 

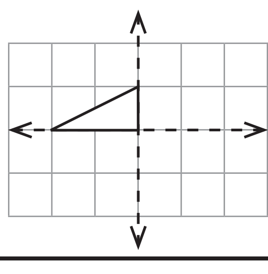
c) 

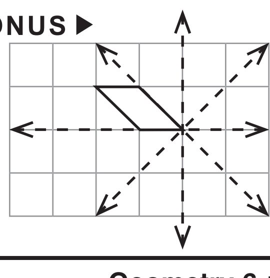
d) 

5. Create a design by reflecting the polygon repeatedly in the mirror lines.

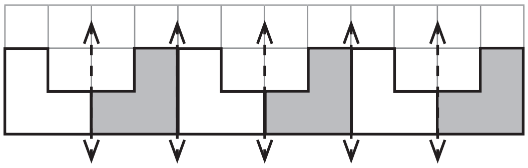
a) 

b) 

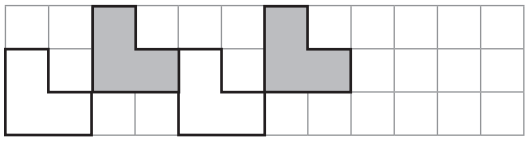
c) 

BONUS ▶


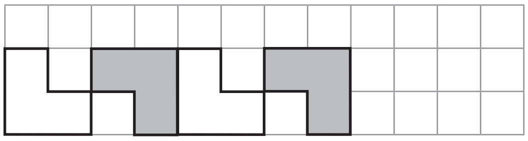
6. The pattern is made by repeating the same type of transformation. Continue the pattern. Identify the type of transformation used. Draw the mirror lines, the translation arrows, or the centres of rotation between each polygon and the next.

a) 

reflection ______________________

b) 

____

c) 

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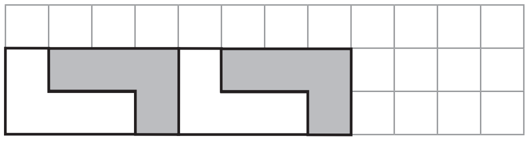
d) 

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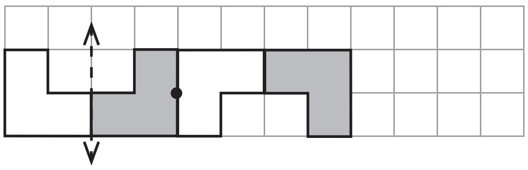
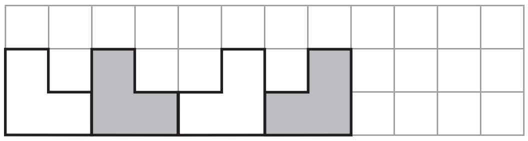
7. Continue the pattern. Describe the transformation that takes each shape to the next.

a) 

1 to 2: *reflection in the vertical line*

2 to 3: *180° CW rotation around marked point*

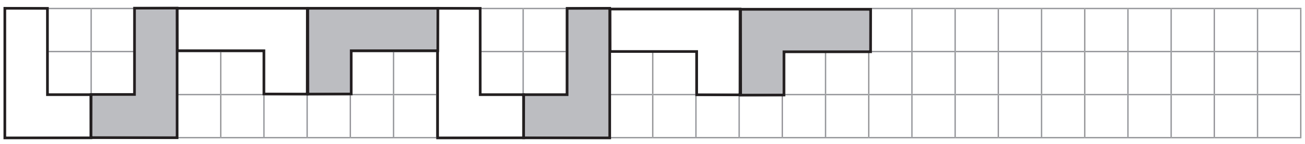
____

b) 

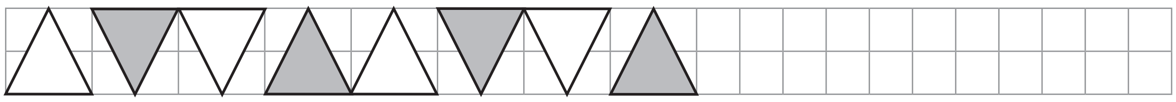
1 to 2: ______________________

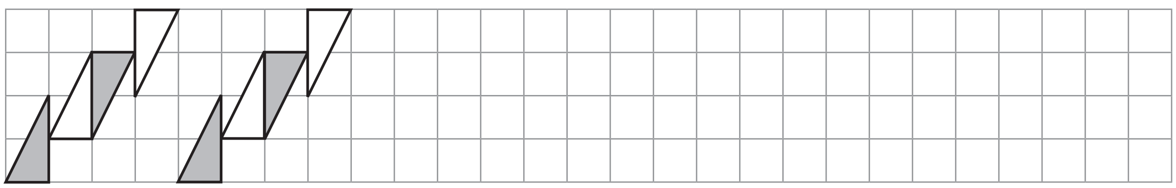
2 to 3: ______________________

____

c) 

1 to 2: ______________________ 2 to 3: ______________________

d) 

e) 

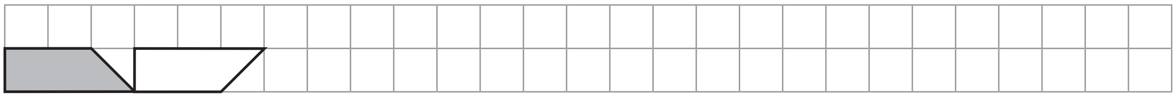
8. a) Which of the patterns in Question 7 can be created using different transformations? Explain.

b) In which parts of Question 7 does the polygon have a line of symmetry?

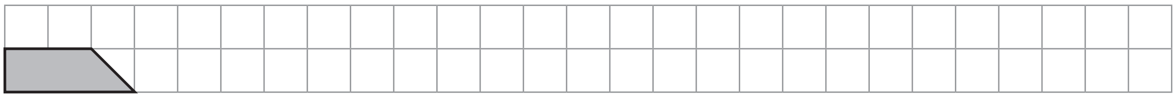
c) What do you notice about the answers in parts a) and b)?

9. Use the polygon and the grid to create a pattern by repeatedly applying the given combination of transformations.

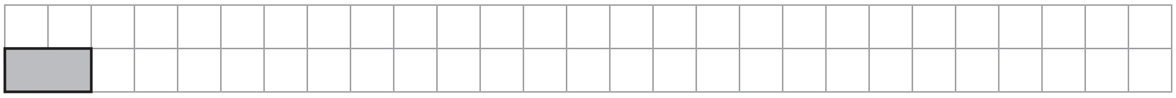
a) Reflect in the top side, then translate 3 units right and 1 unit down.



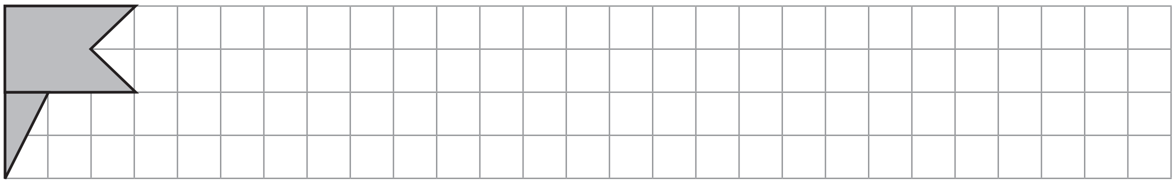
b) Reflect in the top side, then translate 3 units right.
Reflect in the bottom side, then translate 3 units right.



c) Rotate 90° CW around the bottom-right vertex.



d) Reflect in the side common to the polygons, then translate 4 units right.
Reflect in the vertical line through the rightmost vertex or vertices.



10. a) Use grid paper. Draw a polygon that has no lines of symmetry.

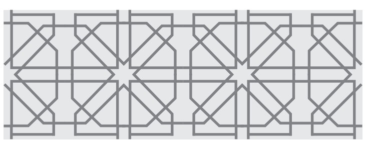
b) Use the polygon you drew in part a) to create a pattern. Use at least two transformations of different types to create your pattern. Describe your pattern.

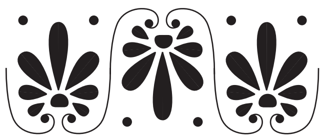
11. a) Use grid paper. Draw a polygon that has a line of symmetry.

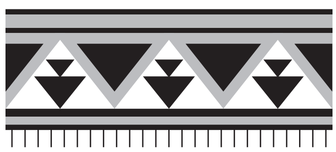
b) Use the polygon you drew in part a) to create a pattern. Use at least two transformations of different types to create your pattern. Describe your pattern.

c) Describe the pattern you created using different transformations. If you cannot, try a different pattern.

12. Draw a rectangle around the smallest part that is transformed to create the pattern. Describe the transformations used to create the pattern.

a) 

b) 

c) 

BONUS ▶ Find a pattern that is made using transformations. Draw the pattern. Describe the transformations used.