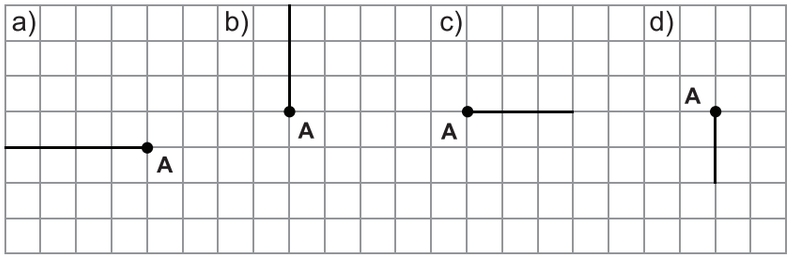
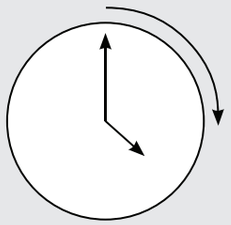
G8-2 Squares on a Grid

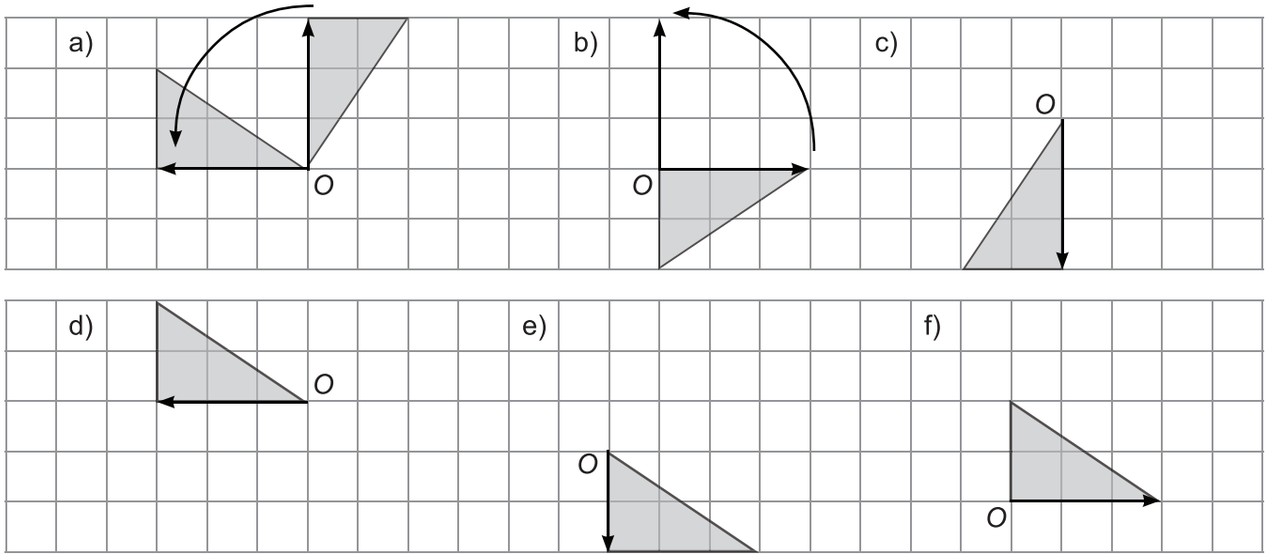
1. Rotate each line segment 90° ($\frac{1}{4}$ turn) clockwise around point A.



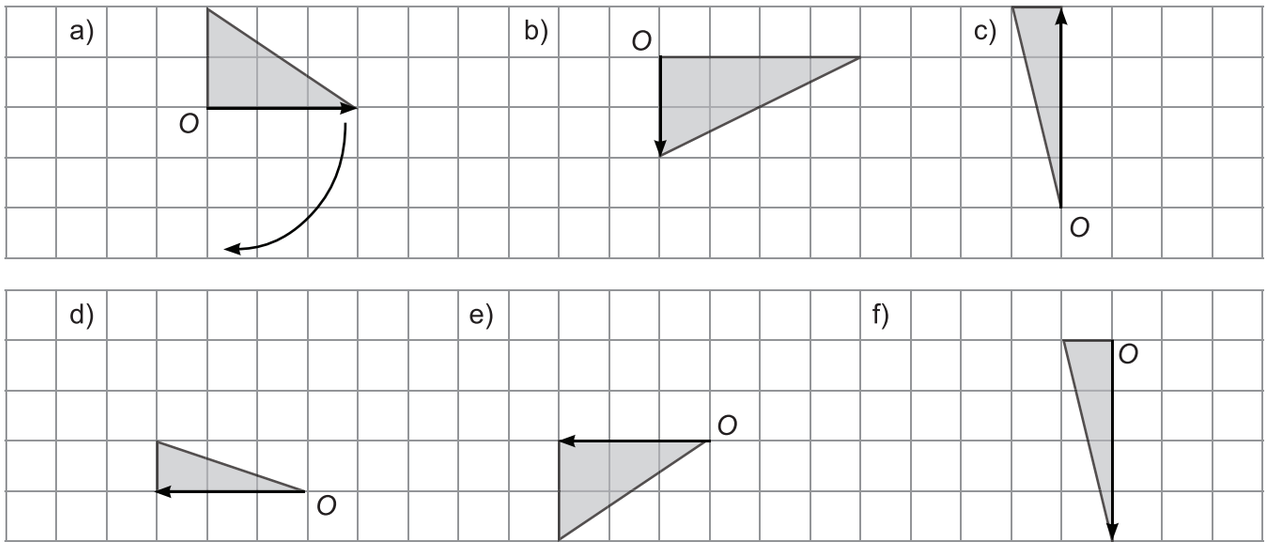
Clock hands turn **clockwise**.



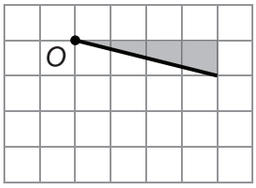
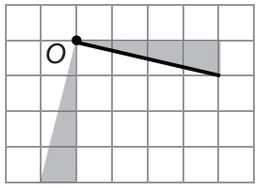
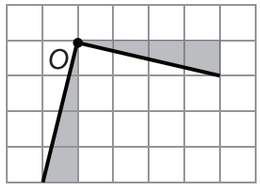
2. Rotate each triangle 90° ($\frac{1}{4}$ turn) counter-clockwise around point *O*. (Rotate the arrow first.)



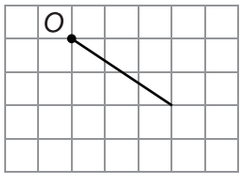
3. Rotate each triangle 90° ($\frac{1}{4}$ turn) clockwise around point *O*. (Rotate the arrow first.)

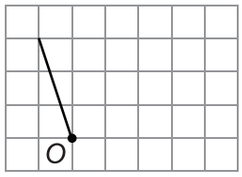


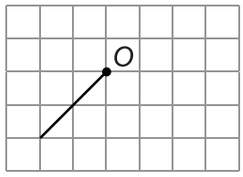
How to rotate a slant line 90° clockwise around *O*

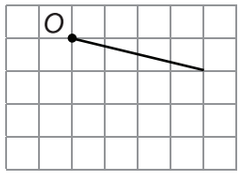
Step 1: Shade a right triangle as shown.	**Step 2:** Rotate the triangle.	**Step 3:** Emphasize the line at 90° to the given slant line.
		

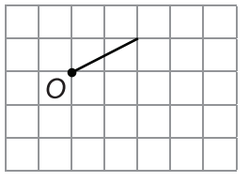
4. a) Rotate these slant lines 90° clockwise around *O*.

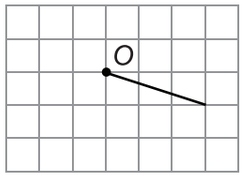
i) 

ii) 

iii) 

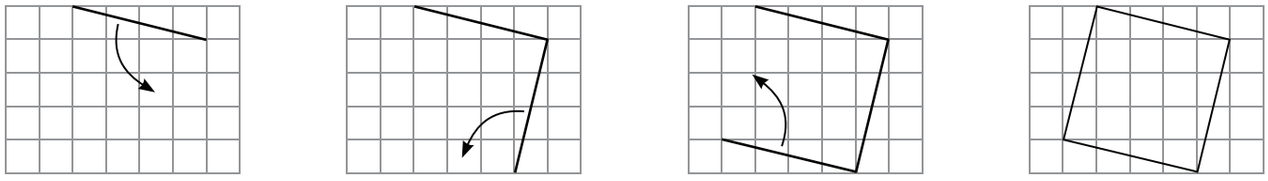
iv) 

v) 

vi) 

b) Use a protractor to check that the angle between the given lines and the rotated lines is 90°.

5. Lucy rotates a line segment 90° counter-clockwise around the endpoints three times.

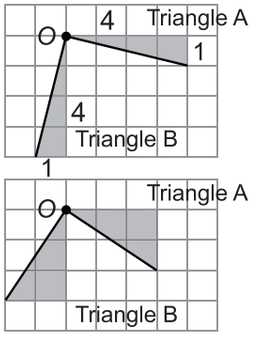


What do you know about the sides and the angles of Lucy's shape? _____

_____

What shape is it? _____

6. Triangle B was made by rotating Triangle A 90° clockwise. Fill in the blanks.



a) Triangle A has horizontal length _4_ and vertical length _1_.

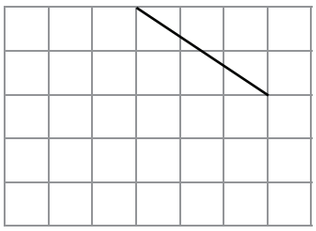
Triangle B has horizontal length _1_ and vertical length _4_.

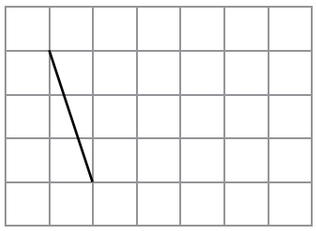
b) Triangle A has horizontal length ___ and vertical length ___ .

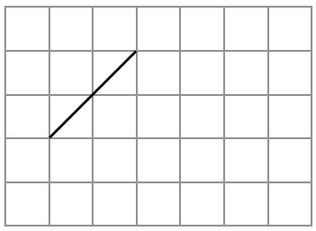
Triangle B has horizontal length ___ and vertical length ___ .

7. Look at your answers in Question 6. If you know the horizontal and the vertical lengths of Triangle A, how can you get the horizontal and the vertical lengths of Triangle B? Check your conjecture with the triangles you drew in Question 4.

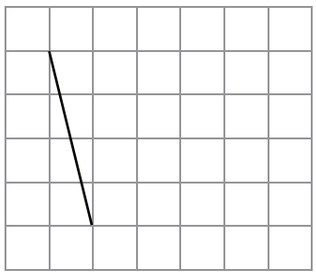
8. Create a square using Lucy's method, starting with the given side. Use your conjecture to rotate the sides 90°.

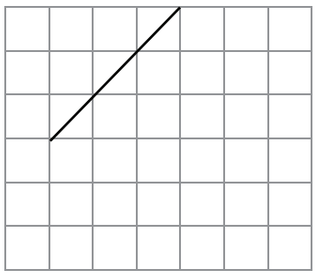
a) 

b) 

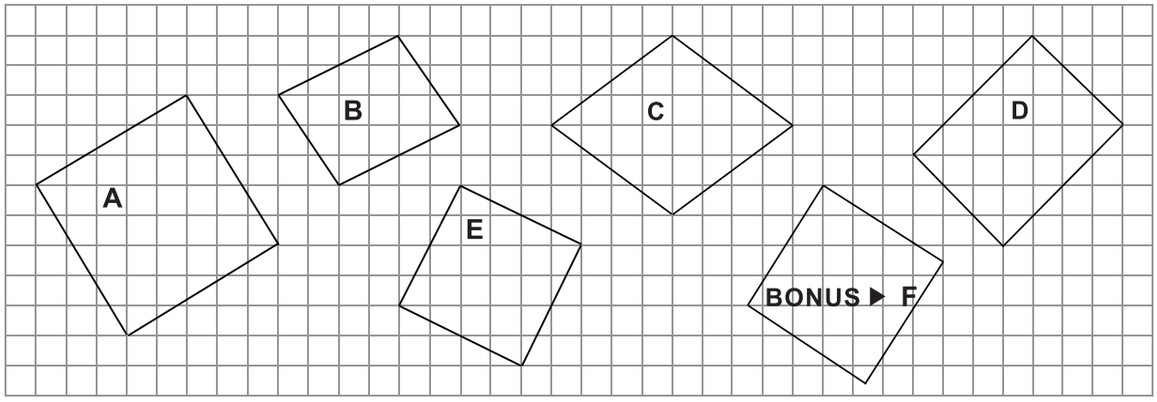
c) 

d) 

e) 

f) 

9. Which of the shapes below are squares? Explain how you know.



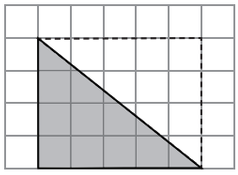
BONUS ▶

a) Which shape in Question 9 is a rhombus but not a square? How do you know?

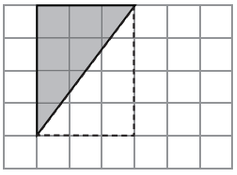
b) Which shape in Question 9 is a rectangle but not a square? Explain.

G8-3 Length of Slant Line Segments

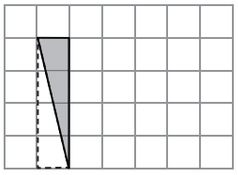
1. Find the area of each right triangle.

a) 

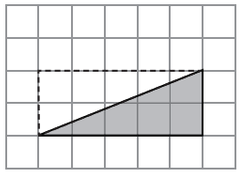
Area = *20* ÷ 2 = *10*

b) 

Area = ___ ÷ 2 = ___

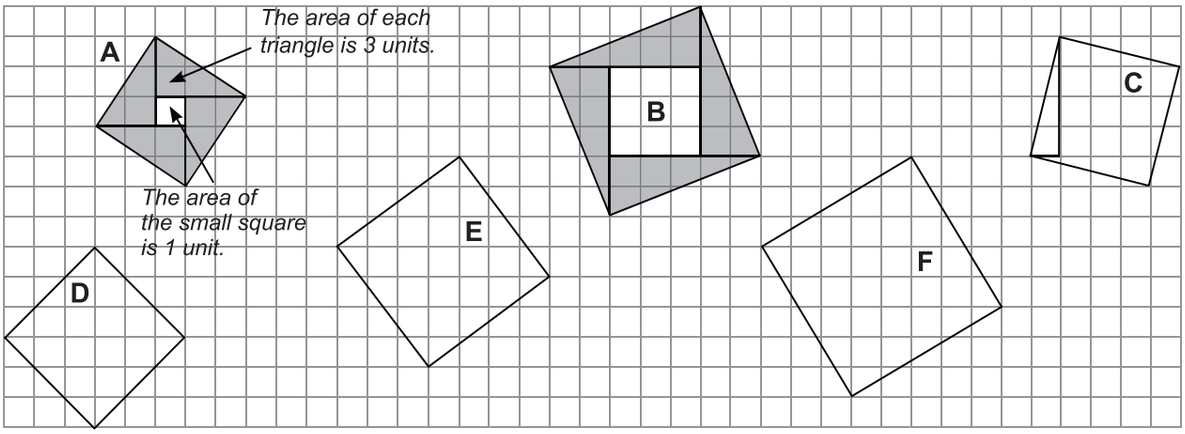
c) 

Area = ___ ÷ 2 = ___

d) 

Area = ___ ÷ 2 = ___

2. Find the area of each square by finding the area of 4 right triangles and what is left.



A Area: 4 × *3* + *1* = *13*

B Area: 4 × ___ + ___ = ___

C Area: 4 × ___ + ___ = ___

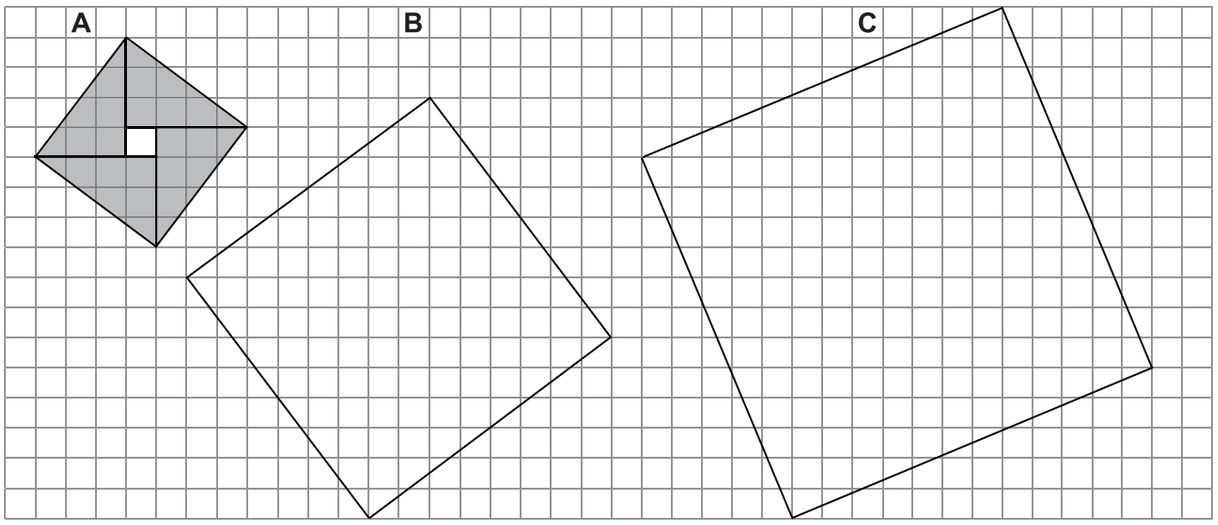
D Area: 4 × ___ + ___ = ___

E Area: 4 × ___ + ___ = ___

F Area: 4 × ___ + ___ = ___

3. What is the side length of square E in Question 2? How do you know?

4. Find the side length of each square.



A side length

$= \sqrt{4 \times 6 + 1}$

$= \sqrt{25} = 5$

B side length

$= \sqrt{4 \times \_\_\_ + \_\_\_}$

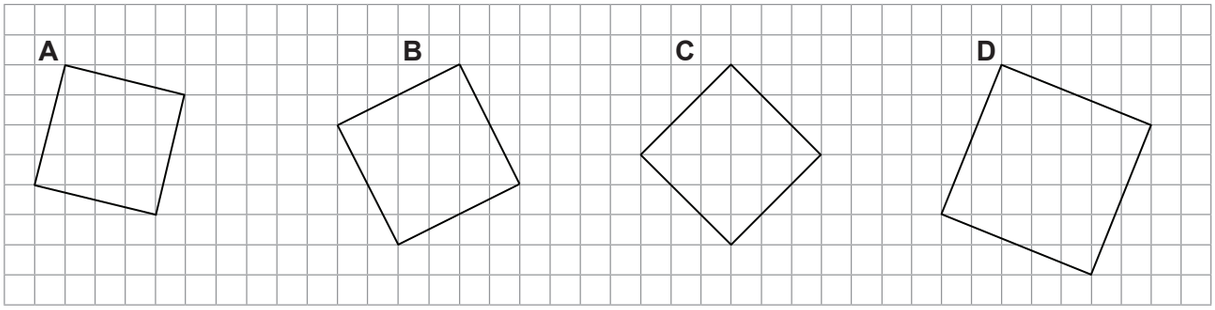
$= \sqrt{\_\_\_} = \_\_\_$

C side length

$= \sqrt{4 \times \_\_\_ + \_\_\_}$

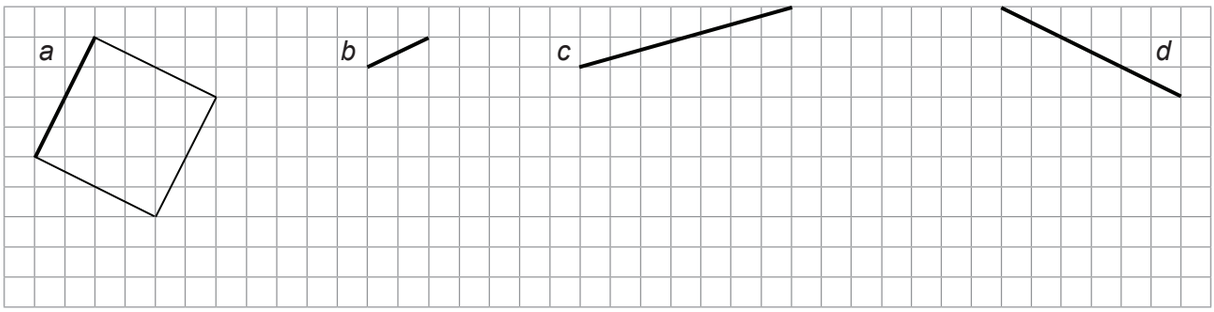
$= \sqrt{\_\_\_} = \_\_\_$

5. Find the side length of each square as a square root. Then estimate the square root.



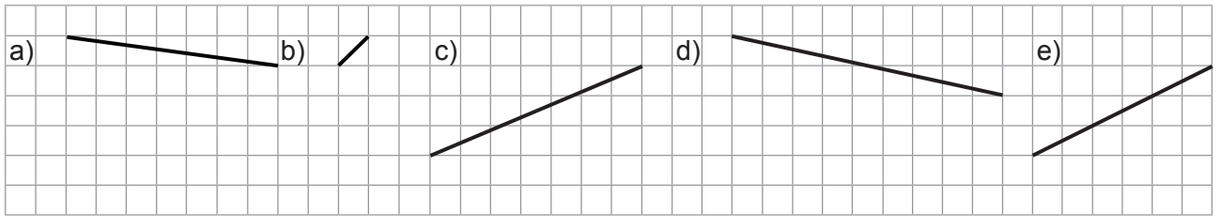
A side length	B side length	C side length	D side length
$= \sqrt{4 \times 2 + 9}$	$= \sqrt{4 \times \_\_\_ + \_\_\_}$	$= \sqrt{4 \times \_\_\_ + \_\_\_}$	$= \sqrt{4 \times \_\_\_ + \_\_\_}$
$= \sqrt{17}$	$= \sqrt{\_\_\_}$	$= \sqrt{\_\_\_}$	$= \sqrt{\_\_\_}$
a little more than *4*	between ___ and ___	a little more than ___	between ___ and ___

6. Draw a square using the line segment as a side and find the area of the square. Then find the length of the line segment.



$a^2 =$ *4 × 4 + 4 = 20*	$b^2 =$ _____	$c^2 =$ _____	$d^2 =$ _____
so $a =$ $\sqrt{20}$	so $b =$ _____	so $c =$ _____	so $d =$ _____

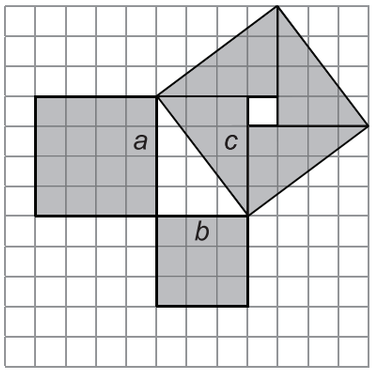
7. Copy these line segments to 1-cm grid paper and find their lengths by drawing squares. First estimate the length to 1 decimal place using the area of the square. Then measure the length with a ruler to check your estimate.



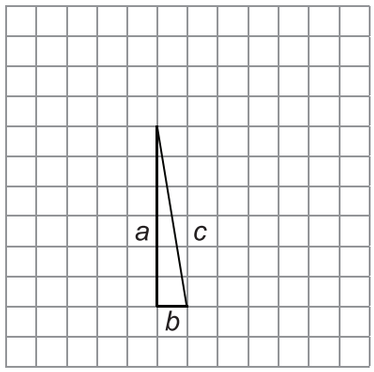
INVESTIGATION ▶

How can you determine the length of the side opposite the right angle in a right triangle?

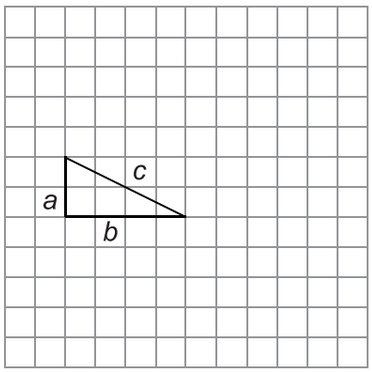
A. Draw squares on all sides of each right triangle. Calculate the areas and fill in the blanks.



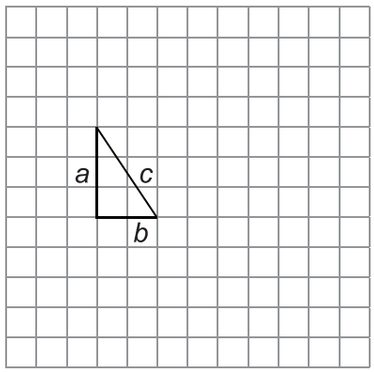
$a^2 =$ 16 $b^2 =$ 9 $c^2 =$ $4 \times 6 + 1 = 25$



$a^2 =$ ___ $b^2 =$ ___ $c^2 =$ _____



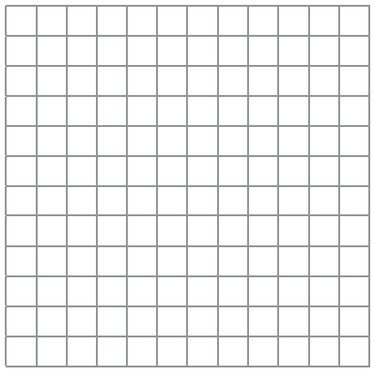
$a^2 =$ ___ $b^2 =$ ___ $c^2 =$ _____

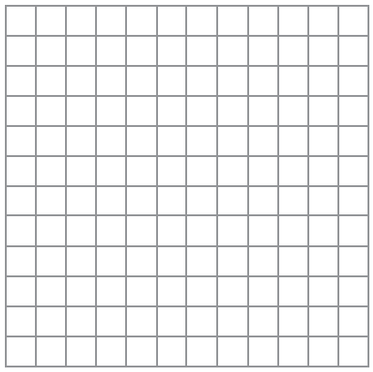


$a^2 =$ ___ $b^2 =$ ___ $c^2 =$ _____

B. Look at the numbers from A. How can you get c^2 from a^2 and b^2? Make a conjecture: If c is the side opposite the right angle in triangle with sides a, b, c, then $c^2 =$ _____

C. Check your conjecture for two right triangles of your choice.

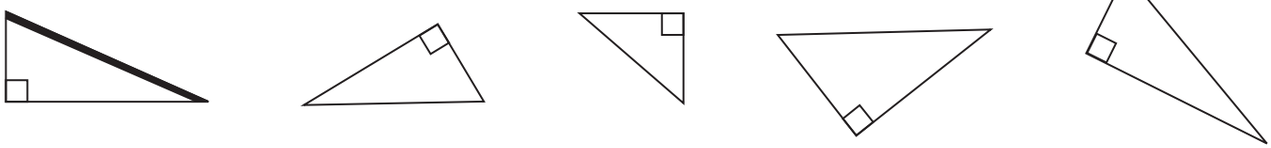




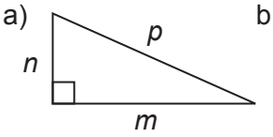
G8-4 The Pythagorean Theorem

Pythagorean Theorem
If a right triangle has sides a, b, c with c opposite the right angle, then $a^2 + b^2 = c^2$.

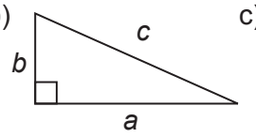
1. Trace the side c according to the Pythagorean Theorem.

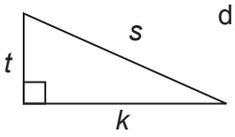


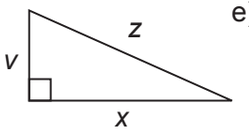
2. What does the Pythagorean Theorem say about each triangle?

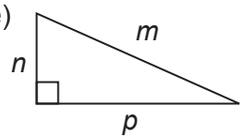
a) 

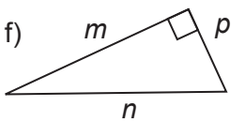
$n^2 + m^2 = p^2$

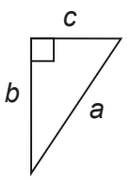
b) 

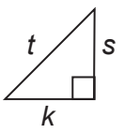
c) 

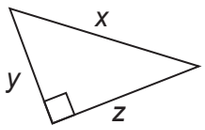
d) 

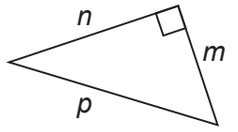
e) 

f) 

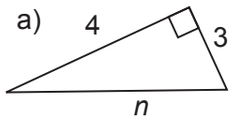
g) 

h) 

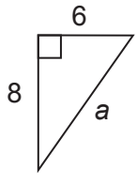
i) 

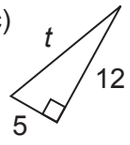
j) 

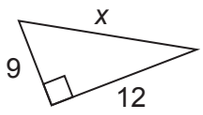
3. Use the Pythagorean Theorem to find the side opposite the right angle.

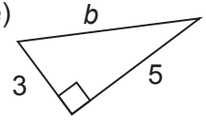
a) 

$n^2 = 4^2 + 3^2$
$= 16 + 9 = 25$
so $n = \sqrt{25} = 5$

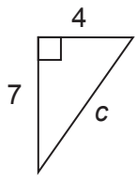
b) 

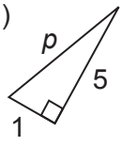
c) 

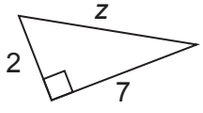
d) 

e) 

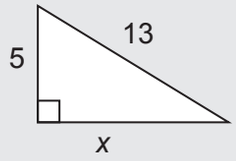
$b^2 = 5^2 + 3^2$
$= 25 + 9 = 34$
so $b = \sqrt{34}$

f) 

g) 

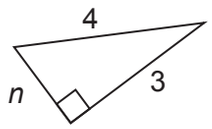
h) 

We can use the Pythagorean Theorem to find any side of a right triangle if two sides are given.



$5^2 + x^2 = 13^2$
$25 + x^2 = 169$
$x^2 = 169 - 25 = 144$
so $x = \sqrt{144} = 12$

4. What does the Pythagorean Theorem say about each triangle? Write an equation, then find the missing side.

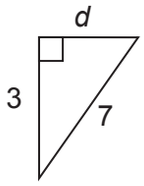
a) 

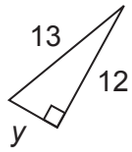
$n^2 + 3^2 = 4^2$

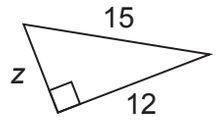
$n^2 + 9 = 16$

$n^2 = 16 - 9 = 7$

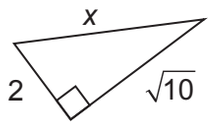
$n = \sqrt{7}$

b) 

c) 

d) 

5. Find the missing side of the triangle using the Pythagorean Theorem. Then estimate the answer using a number line.

a) 

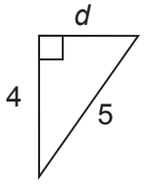
$2^2 + (\sqrt{10})^2 = x^2$

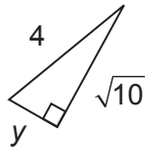
$4 + 10 = x^2$

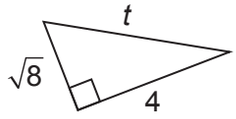
$14 = x^2$

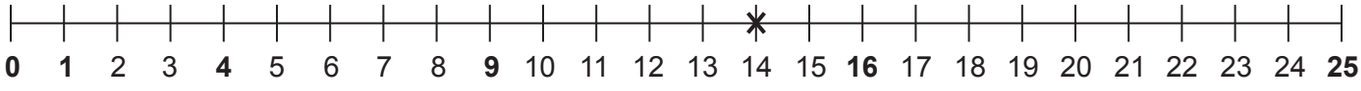
$x = \sqrt{14}$

$x \approx 3.7$

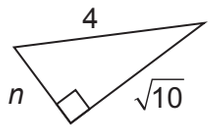
b) 

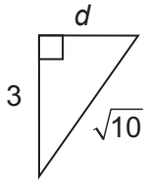
c) 

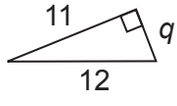
d) 

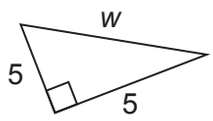


6. Find the missing side of the triangle.

a) 

b) 

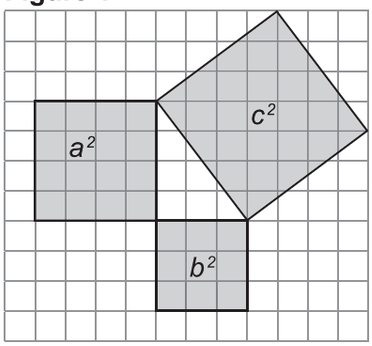
c) 

d) 

G8-5 Proving the Pythagorean Theorem

Pythagorean Theorem
If a right triangle has sides a, b, c with c opposite the right angle, then $a^2 + b^2 = c^2$.

Figure 1



INVESTIGATION 1 ▶ Why does the Pythagorean Theorem work?

A. What does the Pythagorean Theorem say about the areas of squares a^2, b^2, and c^2? _____

_____

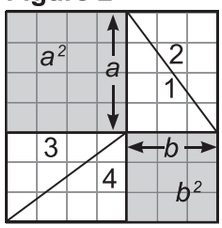
B. Figure 2 shows how to create a large square from the right triangle in Figure 1 (Triangle 1) and the two smaller squares.

Do triangles 1, 2, 3, and 4 all have the same area? How do you know?

_____

_____

Figure 2

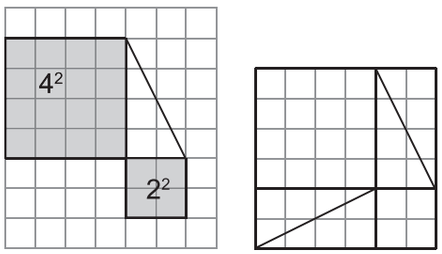


C. Let T represent the area of Triangle 1. Write a formula for the area of the large square using a^2, b^2, and T.

area of large square = _____

D. Create a large square from the smaller squares and four right triangles like the triangle in the figure.

a)



b)

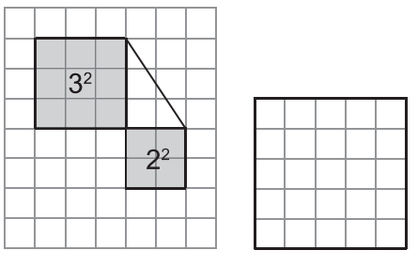
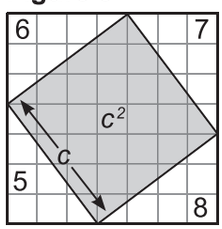


Figure 3



E. Figure 3 shows a different way to create a large square from the right triangle and the largest square in Figure 1.

Do Triangles 5, 6, 7, and 8 in Figure 3 all have the same area? How do you know?

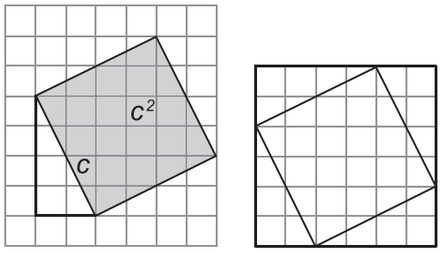
_____

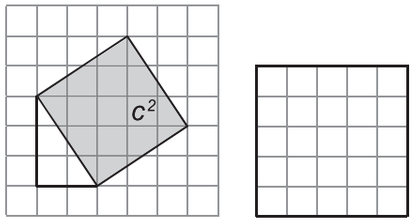
_____

F. Let T represent the area of Triangle 5. Write a formula for the area of the large square in using c^2 and T.

area of large square = _____

G. Create a large square from the square and four right triangles like the triangle in the figure.

a) 

b) 

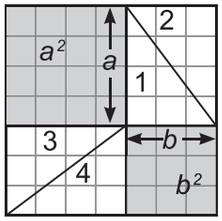
H. Do Triangle 1 in Figure 2 and Triangle 5 in Figure 3 have the same area? _____

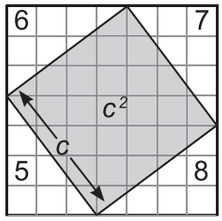
I. Compare the formulas you wrote (for the area of the large squares) in parts C and F. How can you derive the Pythagorean Theorem from these formulas?

_____

_____

_____





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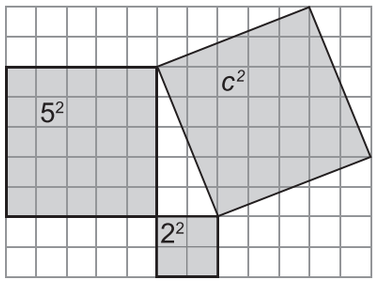
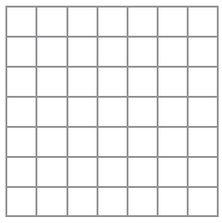
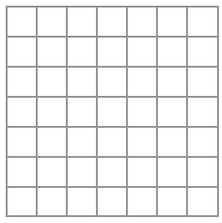
1. a) Prove the Pythagorean Theorem for a triangle with sides $a = 2$, $b = 5$, and c.





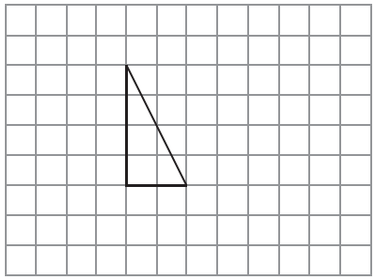
$5^2 + 2^2 = 7^2 - 4 \times$ ___

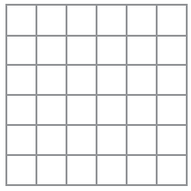


$c^2 = 7^2 - 4 \times$ ___

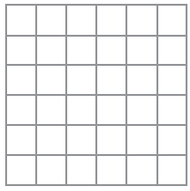
b) Find c using the Pythagorean Theorem: $c^2 =$ ___ + ___, so $c =$ _____.

2. Prove the Pythagorean Theorem for a triangle with sides $a = 4$, $b = 2$, and c. Then find c.





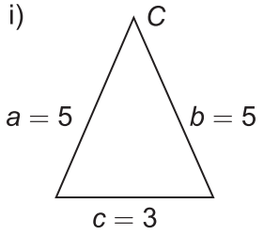
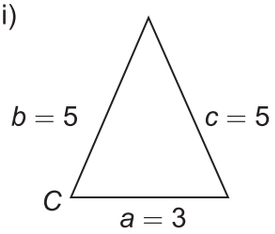
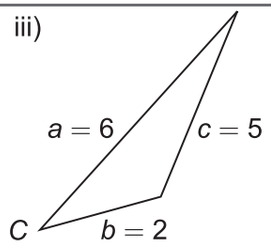
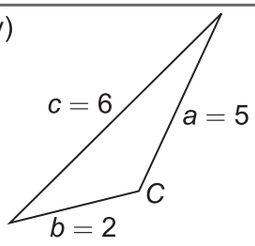
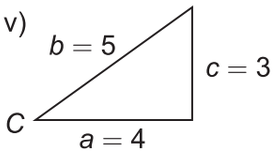
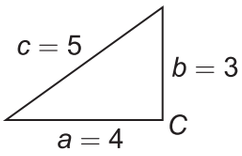
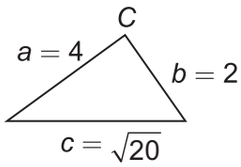
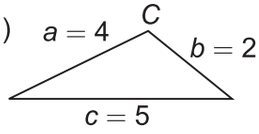
_____



c^2 =_____ =_____

3. Prove the Pythagorean Theorem for a right triangle of your choice.

4. a) In each triangle, $\angle C$ is opposite side c. Find $a^2 + b^2$ and c^2 in your notebook, then look at $\angle C$. Write $<$, $>$ or $=$ in the blanks.

i)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°	ii)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°	iii)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°	iv)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°
v)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°	vi)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°	vii)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°	viii)  c^2 ___ $a^2 + b^2$ $\angle C$ ___ 90°

b) Make a conjecture about a triangle with sides a, b, c and angle C opposite the side c:

When $\angle C < 90°$, then c^2 ____ $a^2 + b^2$,

when $\angle C > 90°$, then c^2 ____ $a^2 + b^2$,

when $\angle C = 90°$, then c^2 ____ $a^2 + b^2$.

c) Draw three more triangles and check your conjecture.

5. a) Can there be a triangle with a right angle and an obtuse angle? ____

b) The Pythagorean Theorem applies to right triangles.
In a right triangle, what is the measure of the largest angle? ____

c) In which of the triangles in Question 4 was C the largest angle?_____

d) Why does the Pythagorean Theorem not work for triangles iv) and viii) in Question 4?

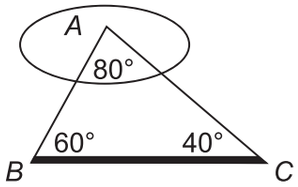
e) Triangle v) in Question 4 is a right triangle. Why does c^2 not equal $a^2 + b^2$?

Hint: Look at your answer in c).

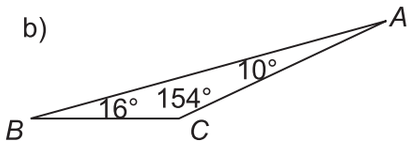
f) Why is triangle v) in Question 4 not a counter-example to the Pythagorean Theorem?

INVESTIGATION 2 ▶ Where is the longest side in a triangle?

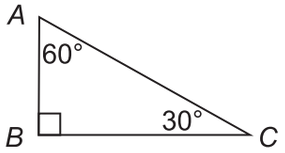
A. Measure the sides of these triangles. Circle the largest angle and trace the longest side.

a) 

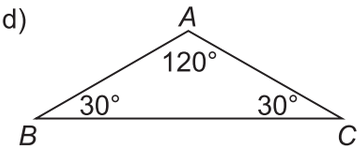
$AB =$ *21* mm
$AC =$ *29* mm
$BC =$ *33* mm

b) 

$AB =$ ____ mm
$AC =$ ____ mm
$BC =$ ____ mm

c) 

$AB =$ ____ mm
$AC =$ ____ mm
$BC =$ ____ mm

d) 

$AB =$ ____ mm
$AC =$ ____ mm
$BC =$ ____ mm

B. Make a conjecture about where the longest side of a triangle is relative to the largest angle:

The longest side of a triangle is always _____ the largest angle.

C. In a right triangle, can there be an angle larger than the right angle? _____

D. Sketch a triangle ABC with $\angle B = 90°$. Which side is the largest?_____

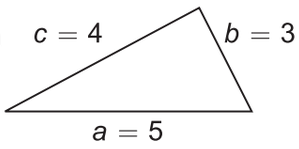
E. Which side is the largest in a triangle DEF with $\angle F = 90°$? Can you tell without sketching $\triangle DEF$?

---

6. Circle the side that is opposite the largest angle in these triangles.

a) 3 cm, 4 cm, 6 cm b) 3 m, 7 m, 5 m c) 1, 2, $\sqrt{5}$ d) 10 km, 10 km, 14 142 m

7. Tegan wants to check whether the triangle at right is a right triangle using the Pythagorean Theorem. She substitutes the lengths of the sides into the formula $a^2 + b^2 = c^2$:



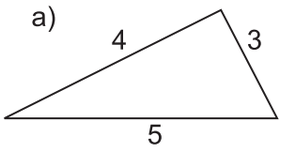
$a = 5$, $b = 3$, $c = 4$, so

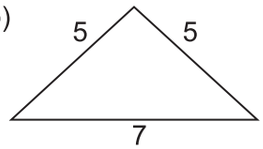
$a^2 + b^2 = 25 + 9 = 34$, and $c^2 = 16$.

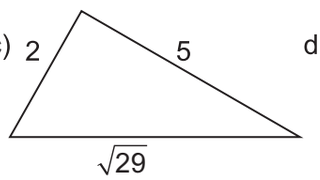
$16 \neq 34$, so Tegan thinks the triangle is not a right triangle. Is she correct? Explain.

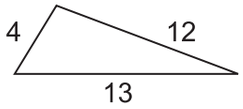
_____

8. Order the side lengths from least to greatest. Then use the Pythagorean Theorem to check whether these triangles are right triangles.

a) 

b) 

c) 

d) 

e) 2, 4, $\sqrt{6}$ f) 1, 2, $\sqrt{3}$ g) 2, 4, $\sqrt{20}$ h) $\sqrt{5}$, $\sqrt{3}$, $\sqrt{2}$ i) 2, $\sqrt{5}$, $\sqrt{3}$

G8-6 Problem-Solving — Using a Formula

When you read a word problem, identify **what you need to find** and **the information you are given**. If you can solve the problem using a **formula**, write the formula.

1. For each of the problems below underline what you need to find and circle the measurements you are given. Then write the formula you are going to use.

a) A parallelogram has (base 5 cm) and (height 35 mm). What is the area of the parallelogram?

Formula: *Area of parallelogram = base × height*

b) A right triangle has short sides 20 cm and 30 cm. What is the length of the longest side?

Formula: _____

c) Find the area of a triangle with base 2 m and height 75 cm.

Formula: _____

For geometric or measurement problems, it often helps **to make a sketch**. Remember, the sketch does not have to be perfect, but it should include all the information you know.

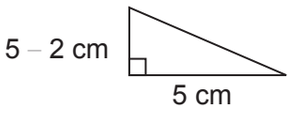
2. **Problem:** The shortest side of a right triangle is 5 cm long. The middle side is 2 cm longer. What is the length of the longest side?

a) Underline what you need to find and circle what you are given.

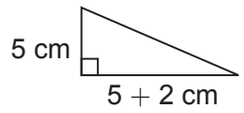
b) Which formula are you going to use? _____

c) Which of these sketches will be the most help in solving the problem? Explain your choice.

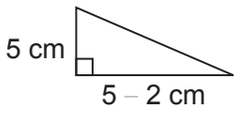
A.



B.



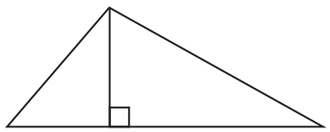
C.



3. For each problem below:
- underline **what you need to find**
- circle **the measurements you are given**
- **label the sketch** to show the information that you know
- write **the formula** you are going to use

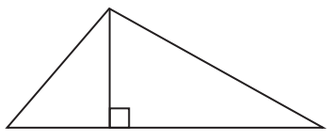
Do not solve the problems yet!

a) In a triangle *ABC*, *BD* is the height, from *B* to the side *AC*. *AD* = 3 cm, *BD* = 5 cm, and *CD* is as long as *AD* and *BD* together. What is the **area** of the triangle?



Formula: _____

b) In a triangle *ABC*, *BD* is the height, from *B* to the side *AC*. *AD* = 3 cm, *BD* = 5 cm and *CD* is as long as *AD* and *BD* together. What is the **length of the side** *AB* of the triangle?



Formula: _____

When you know what formula you are going to use, look for the values that you need in the word problem. Do you have all the measurements you need to use the formula? Sometimes you will need to do a calculation to **find a value that is not given**.

You will solve the following problem in Questions 4 and 5:

Problem: A parallelogram can be cut into a square and two right triangles. The triangles are as high as the square. The base of each triangle is only half as wide as the square. The height of the square is 8 cm. What is the area of the parallelogram?

4. Judy decides to use the formulas for the area of a square and the area of a triangle to solve the problem.

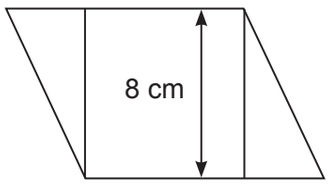
a) Judy is using these formulas:

Area of the square = width × height = height × height

Area of a right triangle = base × height ÷ 2

What information is not given directly in the problem? _____

b) Fill in the values Judy needs and mark them on the sketch.



The height of the square = ____

The height of each triangle = ____

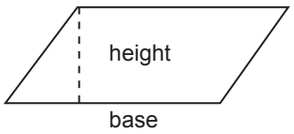
The base of each triangle = ____

c) Area of the square = ____ cm^2

Area of each triangle = ____ cm^2

d) Area of the parallelogram = area of square + ____ × area of each triangle = ____ + ____ = ____

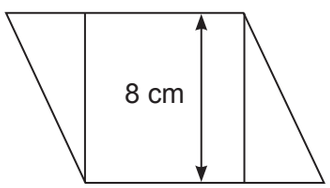
REMINDER▶ area of parallelogram = base × height



5. Guy decides to use the formula for the area of a parallelogram to solve the problem.

a) Which value is not given directly in the problem? ____

b) Fill in the information he needs and mark the information on the sketch.



The base of parallelogram = _____

The height of parallelogram = ____

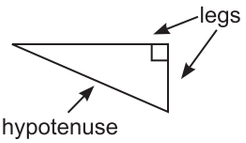
c) Find the area of the parallelogram. Is the answer the same as in Question 4?

When you substitute data into a formula, **make sure that all the units are the same!** If they are not, convert them before you substitute. Do not forget to write the units in the answer.

Remember: $cm \times cm = cm^2$ and $m \times m = m^2$. Also, $\sqrt{m^2} = m$, *and* $\sqrt{cm^2} = cm$

You cannot multiply a measurement in metres by a measurement in centimetres!

6. **Problem:** The short sides of a right triangle are called **legs**. The longest side is called the **hypotenuse**. A right triangle has legs 75 cm and 1 m. What is the length of the hypotenuse?



a) Which units are more convenient—cm or m? Convert all measurements to the units you chose.

leg 1: _____ leg 2: _____

b) Solve the problem.

7. Solve the problems in Question 3.

8. A right triangle has one leg 30 cm shorter than the other. The longer leg is 1.2 m long.

a) What is the area of the triangle?

b) What is the length of the hypotenuse of the triangle?

Sometimes you will not have a formula that will give you what you need to find right away. What you need to find might be **part of a related formula**. Use a variable (such as x) to represent the piece of information you do not know in your formula.

9. **Problem:** The length of a rectangle is twice its width. The perimeter of the rectangle is 120 cm. How long are the sides of the rectangle?

a) Underline what you need to find. Circle the information you know.

b) Perimeter is the sum of the sides of a shape. Write the formula for the perimeter of a rectangle in terms of length (***l***) and width (***w***).

Perimeter of rectangle = _____

c) Here is a sketch for the problem.
Mark the width of the rectangle with an x.
What is the length of the rectangle?

d) Substitute the length and the width into the formula. _____

e) Substitute the value of the perimeter into the formula and solve the equation for x.

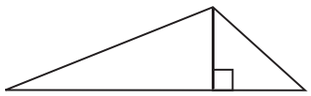
$x =$ ____

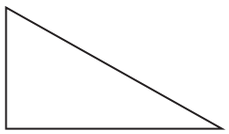
When you **substitute all the data and the variable** into the formula, you get an equation.
For example, in Question 9:

Perimeter of a rectangle: 120 cm width: x cm length: $2 \times x$ or $2x$ cm

Equation: $120 = 2x + x + 2x + x$ or $120 = 6x$

10. For each problem below, mark the information you know on the sketch. Use x for the piece of information that you do not know and mark it on the sketch. Then write an equation.





a) The area of a rectangle is 15 m^2. The short side is 3 m long. What is the long side of the rectangle?

b) The area of a triangle is 3 m^2. The height is 0.5 m long. What is the base of the triangle?

c) The shortest side of a right triangle is 30 cm long. The longest side is 50 cm long. What is the length of the middle side of the triangle?

11. Compare these two problems:

Problem A: A parallelogram has base of 5 cm and area 30 cm^2. What is its height?

Problem B: A parallelogram has height of 5 cm and area 30 cm^2. What is its base?

Do these problems produce different equations? Explain.

12. Fill in the correct units.

a) $3 \text{ cm} \times 3 \text{ cm} = 9$ *cm^2*

b) $9 \text{ cm}^2 \div 3 \text{ cm} = 3$ ____

c) 3 ____ $+ 3 \text{ m} = 6 \text{ m}$

d) 3 ____ $\times 3 \text{ m} = 9 \text{ m}^2$

e) 9 ____ $\div 3 \text{ mm} = 3 \text{ mm}$

f) $\sqrt{(3 \text{ m})^2 + (4 \text{ m})^2} = 5$___

13. Problem: The area of a triangle is 3 m^2. The height is 30 cm. What is the base of the triangle?

a) Would you convert the measurements for the problem into metres or centimetres?_____

b) Convert the measurements into the units you've chosen: Area = _____ Height = _____

c) Substitute the converted units into the formula for the area of a triangle. Use x for the base.

Equation: _____

d) Now solve the equation. Do not forget the correct units in the answer!

14. What is wrong with the following "solutions" to Question 13?

a) The equation is $3 \times x \div 2 = 30$, so $x = 30 \times 2 \div 3 = 20$ m.

b) The equation is $30 \times x \div 2 = 3$, so $x = 3 \times 2 \div 30 = 0.2$ m.

c) The equation is $30 \times x \div 2 = 3$, so $x = 3 \times 2 \div 30 = 0.2$ cm.